

Aircraft Trajectory Prediction in Random Atmosphere

Mathematical background and application to a locally uniform academic case

Christophe Baehr and Cécile Ichard

Météo-France / IMT Toulouse III Paul Sabatier

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METEO FRANCE
Toujours un temps d'avance

What shall we see ?

1. Met Forecasts and Uncertainties.
2. Mathematical formalism for the Aircraft Trajectory Prediction in Random Atmosphere.
3. Real time estimation of the Met and flight parameter probability distributions.
4. A numerical experiment with an academic case.
5. Outcome and further developments.

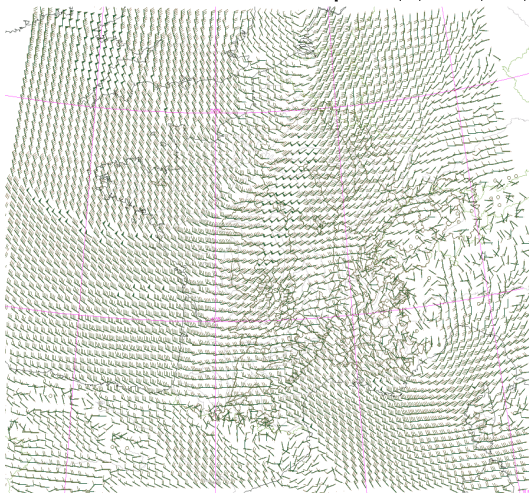
Met Forecasts and Uncertainties

Weather deterministic forecast



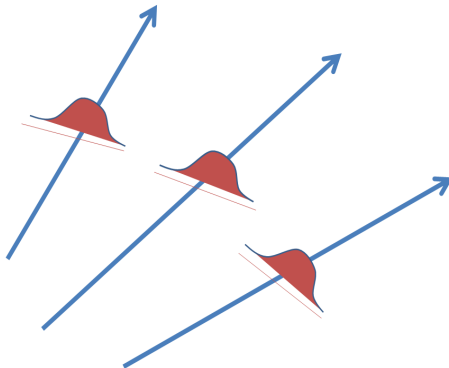
AROME, Run 20/11/2013 00TU

Vent 900 hPa pour le 20/11/13 20TU (ech:20)



How to qualify wind uncertainties?

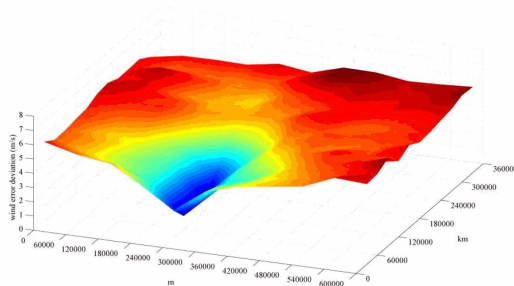
- ⊕ The kernel dressing method.



- ⊕ It uses several hypotheses on the wind field.

Dressing kernel application

⊕ Several hypotheses, but it can provide map of uncertainties.



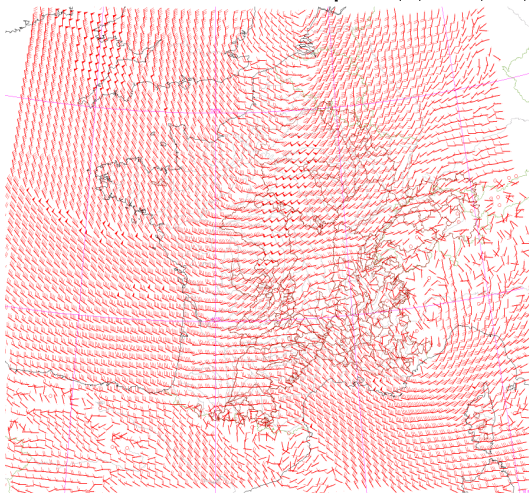
From I. Lympieropoulos and J. Lygeros (2008), Trajectory Prediction by Adaptively Reducing Wind Uncertainty, 10th International Conference on Application of Advanced Technologies in Transportation.

The wind fields are not Gaussian

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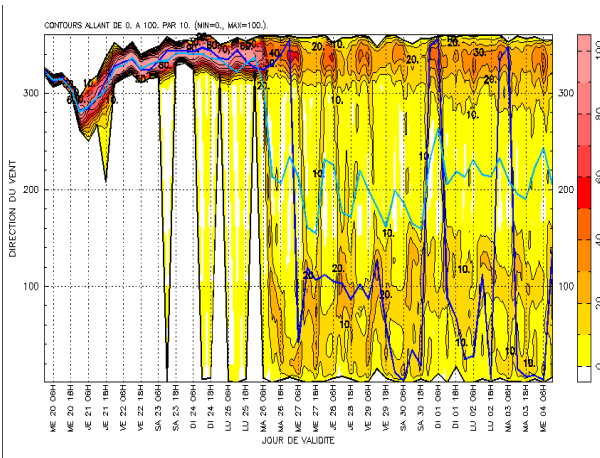
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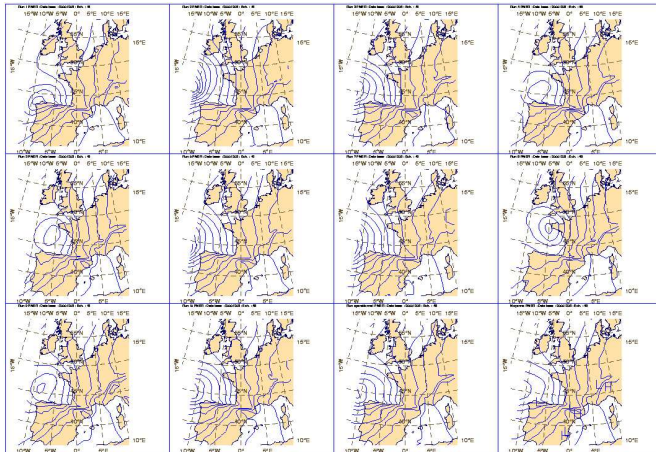
The weather may be multimodal

- ➔ An example of bifurcation with two attractive basins.



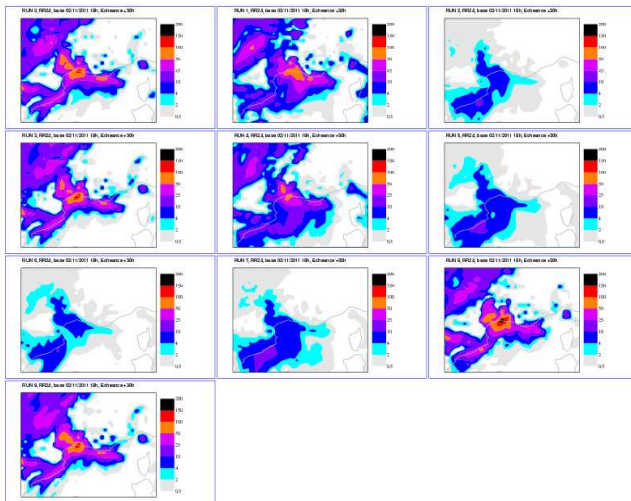
The ensemble Met forecast method

- ⊕ A set of forecasts with different initial conditions.



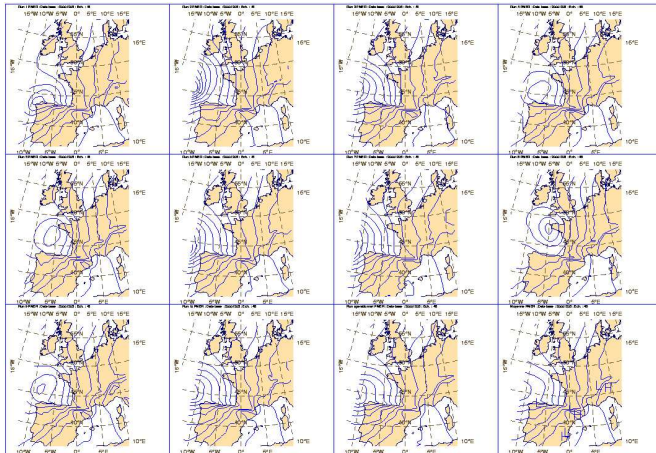
The ensemble Met forecast method

- ➔ An example with deep convection forecasts.



The ensemble Met forecast method

- ⊕ But the members of the ensemble are equidistributed.



Mathematical formalism for the Aircraft Trajectory Prediction in Random Atmosphere

Mathematical formalism

⊖ Let be

⊕ x the location in $E^0 \subset \mathbb{R}^d$, $d = 2, 3$

⊕ X_n^0 the location (trajectory) process of an aircraft in E^0

⊕ $X_{n,x}^1$ the Met field seen as a random medium

⊕ $X_n^2(X_n^0)$ the aircraft flight parameter process for the location X_n^0

⊖ We consider the pair $(X_{n,x}^1, X_n^2(X_n^0))$ and we assume that it is a Markov Process.

⊖ We consider the discrete time stochastic model for X_n^0

$$X_{n+1}^0 = X_n^0 + S_n(X_n^2(X_n^0)) + W_n^f(X_n^0)\Delta t + \Delta X_{n,X_n^0}^1$$

where S_n is the flight strategy and $\Delta X_{n,X_n^0}^1$ is the local Met error.

Mathematical formalism

- ⊖ $X_{n,x}^1$ is supposed to be a Markov chain of transition kernel $M_{n,x}^{(1)}$ with an initial distribution $\eta_{0,x}^{(1)}(dx_{0,x}^{(1)})$.
- ⊖ X_n^2 is a Markov process of transition kernel $M_{x_n^{(1)},n}^{(2)}$ with an initial distribution $\eta_{x_0^{(1)},0}^{(2)}$.
- ⊖ We want to localize the processes to model them in a flight sector or in a homogeneous Met zone in E_n^0 .
- ⊖ The restricted process $(X_{n,l}^2(X_{n,l}^0))$ for the sector D_l has the transition kernel

$$M_{x_{n,l}^{(1)},n,l}^{(2)}(x_{n-1}, dy) = \mathbb{1}_{D_l}(y) M_{x_{n,l}^{(1)},n}^{(2)}(x_{n-1}, dy) + (1 - \mathbb{1}_{D_l}(y)) \delta_{\{\mathcal{U}_n^l\}}(dy)$$

where $x_{n,l}^{(1)}$ is the fixed environment and $\mathcal{U}_n^l$ is an exit point (formally).

- ⊖ We extend the definition of X_n^2 to an entire traffic $(X_{n,l}^{2,i}(X_{n,l}^{0,i}))_{1 \leq i \leq N_n^l}$ where N_n^l is the number of aircraft in the l -zone

Real time estimation of the Met and flight parameter
probability distributions

Learning the pdf of Met and flight parameters

⊕ We consider that we have an Ensemble of Met forecasts. It gives the forecast of the environment $X_{n,x}^1$.

⊕ We consider that we have for each time step radar Mode-S observations : $Y_0 \dots Y_n$.

⊕ We define the probability

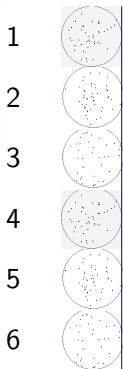
$$\eta_{x_{[0,n],l}^1, n, l}^{(2)} = \mathbb{P}(X_{n,l}^2 \in dx_{n,l}^2 | Y_{[0,n-1],l} = (y_{0,l}, \dots, y_{n-1,l}), \\ X_{[0,n],l}^1 = (x_{0,l}^{(1)}, \dots, x_{n,l}^{(1)}))$$

⊕ In the estimation problem, our interest is on the pair $(X_{n,l}^1, \eta_{x_{[0,n],l}^1, n, l}^{(2)})$

⊕ We have to learn both $X_{n,l}^1$ and $\eta_{x_{[0,n],l}^1, n, l}^{(2)}$. It requires a 2 level algorithm.

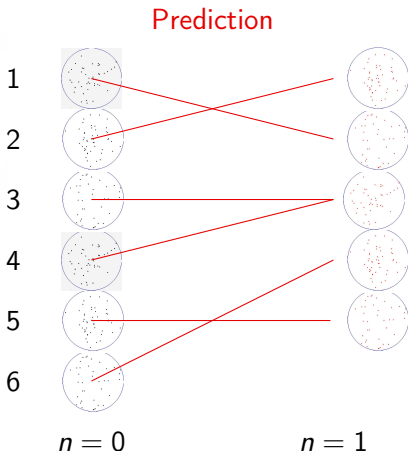
The Particle Islands method

⊕ Each Island is a Met environment, each particle is a TP, the set of Particle sample the TP pdf.



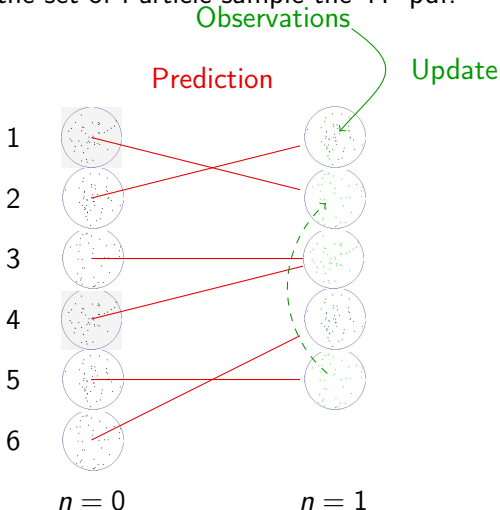
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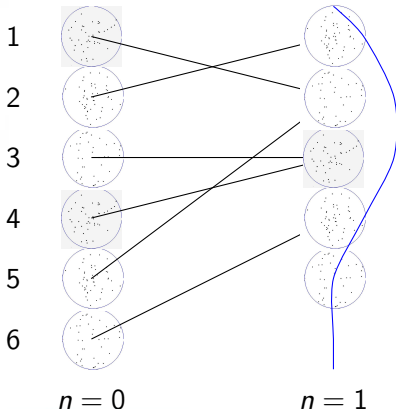
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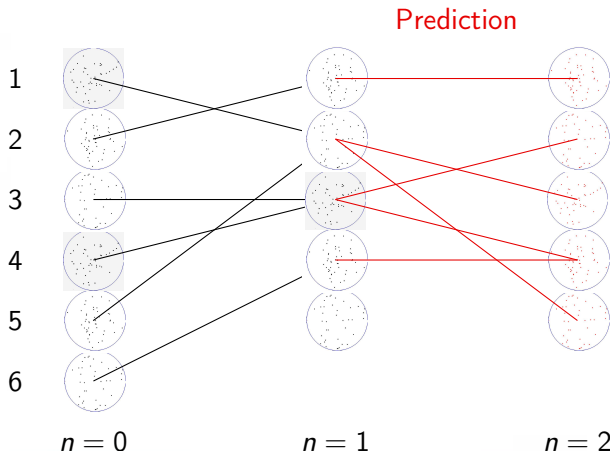
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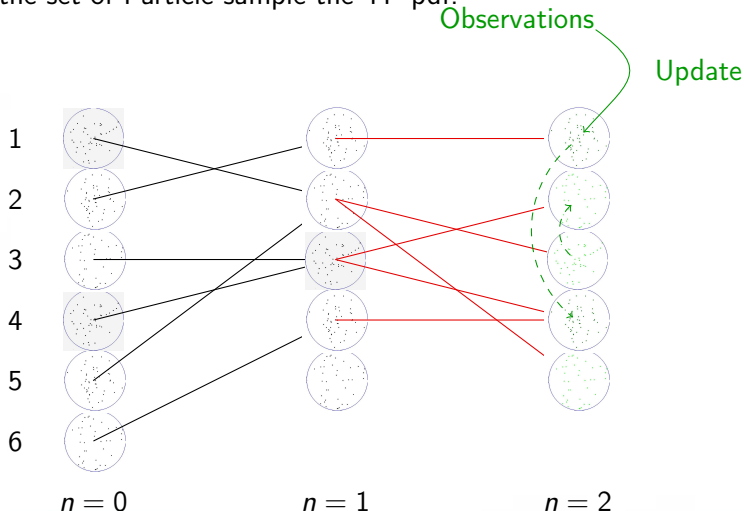
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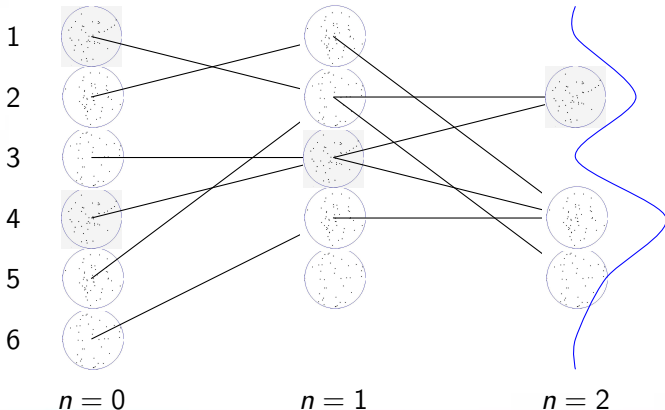
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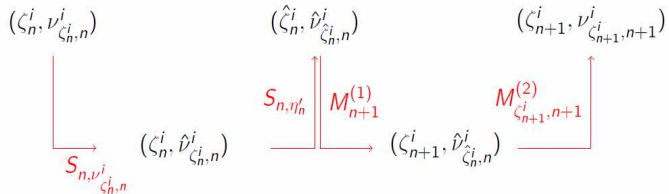
The Particle Islands method

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The (formal) Particle Islands method

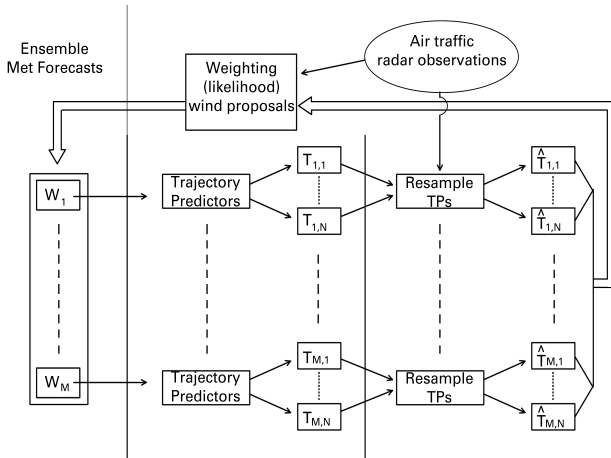
⊕ We define $(\zeta_n^i)_{1 \leq i \leq N}$ the set of Met environment and $(\nu_{\zeta_n^i, n}^i)_{i \leq i \leq N}$ the trajectory pdf approximated by particles in each Island.



where $\eta'_{n,l}(d(x, \nu)) = \eta_{n,l}^{(1)}(dx) \delta_{\eta_{x,n,l}^{(2)}}(d\nu)$, $S_{n,\eta}$ are selection steps, M_{n+1}^1 is the environment prediction and $M_{\zeta_{n+1}^i, n+1}^2$ is the trajectory prediction.

The particle Island algorithm

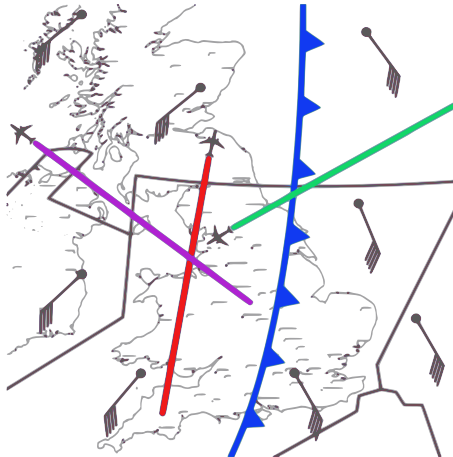
- ⊖ Sequential estimation of Met ensemble weight and Aircraft trajectory flight parameters.



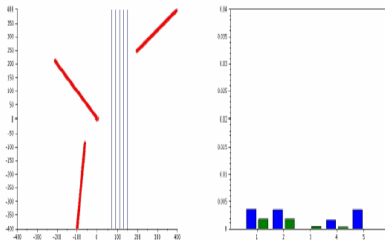
A numerical experiment with an academic case

Description of the numerical experiment

- ⊕ An example of Met configuration with 3 aircraft.

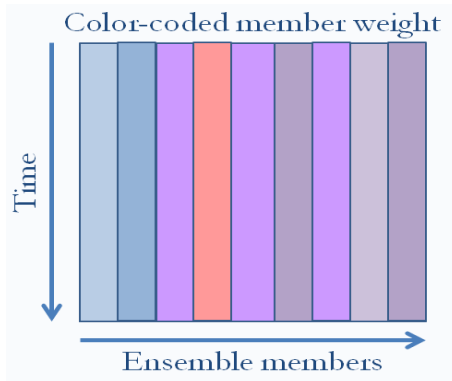


Likelihood evolution of limit proposals



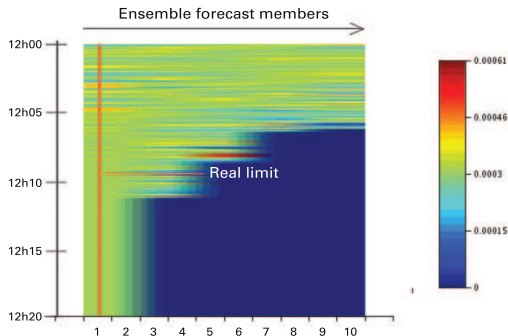
Met Ensemble weight and flight parameter estimations

- ⊖ The following diagrams give with color-coded figures, the time evolution of the Met ensemble weights.



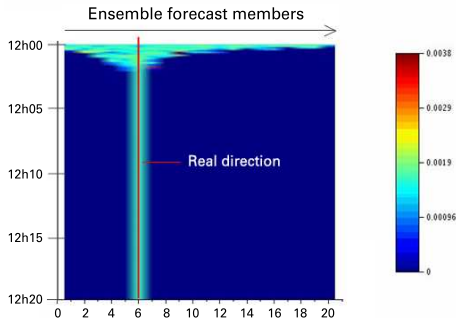
Met Ensemble weight and flight parameter estimations

- ⊕ Learning of the most probable limit between the 2 areas.



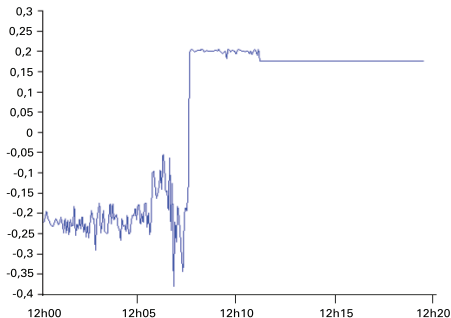
Met Ensemble weight and flight parameter estimations

- ⊕ The evolution of the weights according to the wind direction in the left zone.



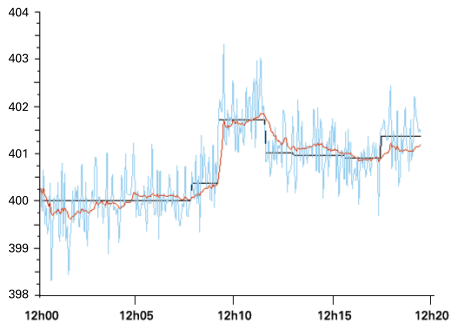
Met Ensemble weight and flight parameter estimations

⊕ Evolution of the relative errors wind force estimation in the left zone.



Met Ensemble weight and flight parameter estimations

⊕ Learning of the TAS for the aircraft # 2.



Outcome and further developments

Outcome ...

- ⊕ We have developed a Mathematical formalism for the Aircraft Trajectory Prediction in Random Atmosphere based on Random Processes in Random Media.
- ⊕ We have applied this modelling to the (real-time) estimation of the likelihood of the Ensemble Met Forecasts AND the joint estimation of flight parameters.
- ⊕ There is no special hypotheses on the random processes. The unique limitation concerns the computational costs.
- ⊕ We have applied the designed algorithm to an academic case with good results.

... and further developments

- ⊕ We have to complete the Mathematical background to take into account all the aspects of the problem and study the algorithm errors.
- ⊕ This work is a preliminary work to the WP-E IMET research program.
- ⊕ We have to use in the IMET context more realistic TP with the NLR algorithm.
- ⊕ Then we intend to propose 4D trajectory optimization according the Met environment and learned flight parameters.